July, 2013

Measuring the Dynamic Range of a Nanometrics Accelerograph

The dynamic range of an accelerometer or accelerograph (an accelerometer with a built-in digitizer) can be measured and plotted as a function of frequency, by comparing the amplitude of instrument self-noise to the clip-level amplitude at specific frequencies. This technical note outlines an appropriate procedure, and is consistent with the methodology used by the U.S. Geological Survey (USGS) in measuring accelerometers to determine if they comply with Advanced National Seismic Systems (ANSS) specifications. The derivation of the formulae is discussed in a theory section. The final section shows plots derived from an actual example data set recorded from a TitanSMA accelerograph to illustrate the procedure.

Procedure

Overview

The first step is to record the self-noise of the instrument. This is most easily done by recording the instrument output directly, providing the site is sufficiently quiet that site noise does not exceed the instrument self-noise. That is more likely to be the case in the high clip level (4g or 2g) modes which have the highest self-noise, but the lower self-noise of the more sensitive ranges is likely to be obscured by site noise.

If a site quiet enough for direct measurements is not available, it is then necessary to set up three or more instruments side-by-side on a single pier in a reasonably quiet environment, and use coherence analysis to produce a power spectral density plot of the non-coherent signal for each instrument. The non-coherent signal is presumed to be instrument self-noise, the signal common to all instruments (the coherent signal) having been removed. For this to be the case, the instrument s must be installed to be mutually aligned and properly leveled so that the X, Y and Z axes are all in the same directions, and care taken to be sure the instruments are well-coupled to the common pier so they all experience the same motion.

The second step is to calculate the power spectral density as a function of frequency, convert that to equivalent root-mean-square (RMS) amplitudes in half-octave bands, and compare with RMS amplitudes of a maximum sine wave output. The difference between the amplitudes of the maximum signal and the self-noise signal is the dynamic range.

For the direct recording of a single instrument, if the instrument self-noise is obscured by site noise for some frequency band but clearly visible for frequencies above and below this band, it may be possible to interpolate the self-noise into the region of high site noise to estimate the dynamic range in this region.

Detailed Procedure

The procedure that follows is for a Titan SMA, Titan EA or Titan XT accelerograph, in which the dynamic range of the combined sensor and internal digitizer is being measured. (To measure the dynamic range of a Titan accelerometer using an external Centaur digitizer, a similar procedure is followed except that the input range of the digitizer is set to 40 Vpp when measuring the clip level, and to 2 Vpp when recording self-noise, so as to ensure the digitizer self-noise is lower than the accelerometer self-noise.)

1) Setup and Recording Self-Noise

At a quiet site, set up the Titan SMA accelerograph. In the Configuration → Accelerometer page, set the accelerometer to 4g mode. In the Configuration → Digitizer page, set the sample rate to 250 Hz, enable DC removal, and set DC removal cut-off to 0.001 Hz. This will provide an output signal band limited from 0.001 Hz (1000 seconds) to about 100 Hz. Be sure the unit and site are undisturbed, shielded from temperature variations and air currents, and let the unit run for at least an hour to come to temperature equilibrium. Next collect at least an hour of data and review the waveform to be sure there are no seismic or cultural noise events present in the data.

2) Calculating Noise Amplitude and Dynamic Range

Create a power spectral density plot from at least 30 minutes of the collected data, plotted in units of dB relative to $\frac{m^2/_{s^4}}{Hz}$. Now convert this to a plot of "RMS amplitude per half-octave bandwidth" (in units of dB relative to m/s^2) using the following formula:

$$Noise\_Amp(f) = PSD(f) - 4.6\ dB + 10log_{10}(f)$$

Where f is the frequency in Hz, and $PSD(f)$ is the power spectral density function plotted from the data collected. This plot now represents the RMS amplitude of the noise signal over a band of one-half octave centered at a given frequency, and can then be directly compared to the RMS amplitude of the clip level.

On this plot, also draw a horizontal line at 29.3 dB, which is the RMS amplitude of the maximum sine wave output of the Titan SMA in 4g mode (see below for derivation). The difference between this clip level line and the $Noise\_Amp(f)$ line is the dynamic range for a given frequency.

Alternatively, the dynamic range can be plotted directly:

$$Dynamic\_Range\ (f) = 29.3\ dB - PSD(f) + 4.6\ dB - 10log_{10}(f)$$

3) Interpretation in the Presence of Site Noise

If self-noise was not measured by using the coherence method with multiple instruments on a single pier, the PSD plot may show site noise exceeding instrument self-noise in some frequency bands, which if not recognized would lead to a measurement of dynamic range that is incorrectly low. If excessive site noise is limited to a frequency band with good self-noise data at frequencies above and below that band, it may be possible to roughly interpolate the self-noise into the region of high site noise. This is possible because the instrument self-noise PSD has a characteristic smooth shape like a bowl.

If the site noise is too high for the PSD to produce the characteristic bowl-shape self-noise plot, a quieter site or use of the coherence method to measure true instrument self-noise is required.

Theory

Comparing the amplitude of the maximum clip level signal (or any signal) to noise floors (or any random process measured by computing power spectral density) requires conversion of the power spectral density (signal power per unit of bandwidth) to equivalent amplitudes. The principle is to determine, for a signal that has a certain power spectral density value at a given frequency, what the RMS (root-mean-square) amplitude of that signal would be if it was bandpass filtered to a specific bandwidth centered around that frequency. It is common to use a ½ octave bandwidth for this calculation, but other bandwidths can be used. For example, if the power spectral density of a noise signal at 10 Hz was $10^{-12}\frac{m^2/s^4}{Hz}$, we ask: what would be the RMS amplitude of a signal having that power density over a ½ octave centered at 1 Hz. This is in the same units as the signal output, in this case units of acceleration: m/s^2, which means it can be compared directly to other output signals such as the amplitude of a just-clipping sine wave.

The upper and lower band limits of a band centered on a given frequency f are:

$$f_{upper} = \sqrt[2n]{band} \cdot f_{center}$$

$$f_{lower} = \frac{1}{\sqrt[2n]{band}} \cdot f_{center}$$

$$bandwidth = f_{upper} - f_{lower} = \left(\sqrt[2n]{band} - \frac{1}{\sqrt[2n]{band}}\right) \cdot f_{center}$$

where $band$ is 2 for octave bandwidths (the upper frequency being twice the lower), or 10 for decade bandwidths (the upper frequency being 10 times the lower), and where n sets the fraction of the band desired, such as 2 for a half octave, 3 for one-third octave, or 6 for one-sixth decade. We select $band = 2$ and $n = 2$ for half-octave, so we have

$$bandwidth\ (in\ Hz) = \left(\sqrt[4]{2} - \frac{1}{\sqrt[4]{2}}\right) \cdot f_{center}$$

The signal power is proportional to the bandwidth if the power spectral density is constant across the band, so the signal power (square of the RMS amplitude) is the power spectral density multiplied by the bandwidth:

$$(Noise\_amp(f))^2 = PSD(f) \cdot bandwidth$$

or:

$$Noise\_amp(f) = \sqrt{PSD(f)} \cdot \sqrt{bandwidth}$$

to convert to dB, take $20log_{10}$ of each side:

$$20log_{10}(Noise\_amp(f)) = 10log_{10}(PSD(f)) + 10log_{10}\left(\sqrt[4]{2} - \frac{1}{\sqrt[4]{2}}\right) + 10log_{10}(f_{center})$$

finally in dB: $$Noise\_amp_{dB}(f) = PSD_{dB}(f) - 4.6 + 10log_{10}(f_{center})$$

The dynamic range at a given frequency is then the difference of the RMS amplitude of a maximum signal, such as a sine wave with peak amplitude at the clip level of the instrument, and the RMS amplitude of the instrument's self-noise within a half-octave band centered at that frequency.

$$Dynamic\_Range_{dB}(f) = Max\_signal - PSD_{dB}(f) + 4.6\ dB - 10log_{10}(f)$$

The RMS amplitude of the maximum output sine wave signal is calculated as follows: the sensitivity of the Titan SMA in 4g mode is 0.5 µg per single count of the internal 24-bit digitizer, which has a peak clipping value of $\pm 2^{23}$ counts. The clip level of the Titan SMA in 4g mode is therefore $2^{23} \times 0.5\ \mu g = 4.19\ g$, or $4.19 \times 9.81\ \frac{m}{s^2} = 41.1 \frac{m}{s^2}$. The RMS amplitude of a sine wave with that peak amplitude is obtained by dividing by $\sqrt{2}$ and is $29.1 \frac{m}{s^2}$, which in dB is $20log_{10}(29.1) = 29.3\ dB$. The maximum amplitude of the Titan SMA can be readily verified by shaking the instrument to exceed 4g acceleration and observing the maximum values produced.

So for TitanSMA in 4g mode: $Dynamic\_Range_{dB}(f) = 29.3\ dB - PSD_{dB}(f) + 4.6\ dB - 10log_{10}(f)$

For a TitanSMA in 2g mode, the sensitivity is 0.25 µg per count, the RMS amplitude of a clipping sine wave is $14.5 \frac{m}{s^2}$ or 23.3 dB, and $Dynamic\_Range_{dB}(f) = 26.3\ dB - PSD_{dB}(f) + 4.6\ dB - 10log_{10}(f)$

For the example of a noise signal with a power spectral density of $10^{-12} \frac{m^2/s^4}{Hz}$ $(-120\ \text{db})$ at 10 Hz:

$$Noise\_amp_{dB}(10Hz) = 10log_{10}(10^{-12}) - 4.6 + 10log_{10}(10\ Hz) = -114.6\ dB$$

and the dynamic range at that frequency would be:

$$Dynamic\_Range_{dB}(10\ Hz) = 29.3\ dB - (-114.6\ dB) = 143.9\ dB$$

See Bormann, P., & Wielandt, E. (2013). Seismic Signals and Noise. In P. Bormann (Ed.), *New Manual of Seismological Observatory Practice 2 (NMSOP2)* (pp. 1-62). Potsdam: Deutsches GeoForschungsZentrum GFZ. doi:10.2312/GFZ.NMSOP-2_ch4. for more in-depth theory this method is based on.

Example

Figure 1 shows a PSD plot of 30 minutes of data collected from a TitanSMA installed on a granite slab at Nanometrics manufacturing facility in Kanata, Ontario Canada. This site is located on some 60 feet of clay over bedrock, situated in a suburban technology park with residential, office and light manufacturing facilities. High frequency cultural noise amplified by the geology makes this a somewhat noisy site. The data was collected in the early morning hours (about 4:00 AM) while road traffic was minimal. The graph shows the signal power spectral density in dB for the vertical channel (Z) and one horizontal channel (Y), plotted together with the New High Noise Model (NHNM) and New Low Noise Model (NLNM) curves for reference. The X horizontal channel is omitted for clarity as it is similar to the Y channel. This was a fairly quiet time for this area, but high frequency site noise exceeding the instrument self-noise is plainly evident in the 2-90 Hz band. We can see that the instrument self-noise dominates below 2 Hz. The site noise falls below the instrument self-noise at 90 Hz on the horizontal channel but still dominates on the vertical channel. So for horizontal channels (but not the vertical), the narrow band from 90-100 Hz is also largely instrument noise. Because the TitanSMA self-noise is shaped like a bowl, it is possible to estimate by interpolation what the instrument self-noise is likely to be for the horizontal channel in the 2-90 Hz band, but for a definitive result in the 2-90 Hz band the two options would be to test at a quieter site or to use coherence analysis to subtract site noise from the signal.

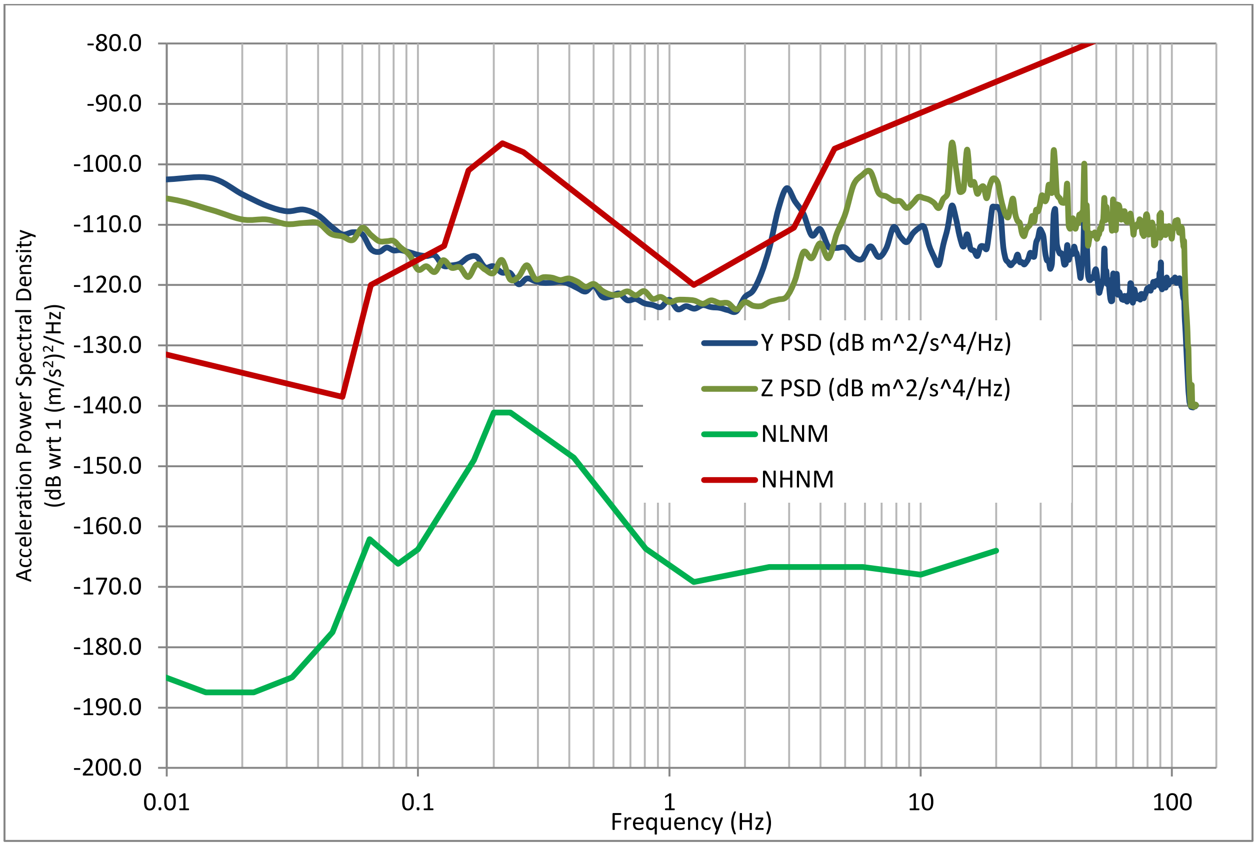

Figure 1- Power Spectral Density of TitanSMA Output Signals with New Low and High Noise Models for Reference
Start time: 2013-07-25 08:00 UTC, Duration 0:30:00
Number of FFT Windows: 17 Smoothing: 30 bins/decade up to 100 points/bin

Figure 2 plots $Noise\_Amp_{dB}(f)$ (the RMS noise amplitude in ½ octave bands) together with the 29.3 dB RMS maximum sine wave level. If the noise plot was just instrument self-noise, the difference between the maximum sine wave level and the noise line would be the dynamic range. This is true for frequencies below 2 Hz for both vertical and horizontal, and also for 90-100 Hz on the horizontal, but in this case site noise dominates both vertical and horizontal channels in the 2-90 Hz band.

Figure 3 plots $Dynamic\_Range_{dB}(f)$, the difference between $Noise\_Amp_{dB}(f)$ and the RMS maximum sine wave. We see that the TitanSMA dynamic range is better than 155 dB for the frequency range below 2 Hz, and better than 132 dB at 100 Hz on the horizontal channel. Site noise above 2 Hz on the vertical channel and in the 2-90 Hz band on the horizontal channel hides the instrument self-noise and prevents seeing the actual dynamic range in these regions.

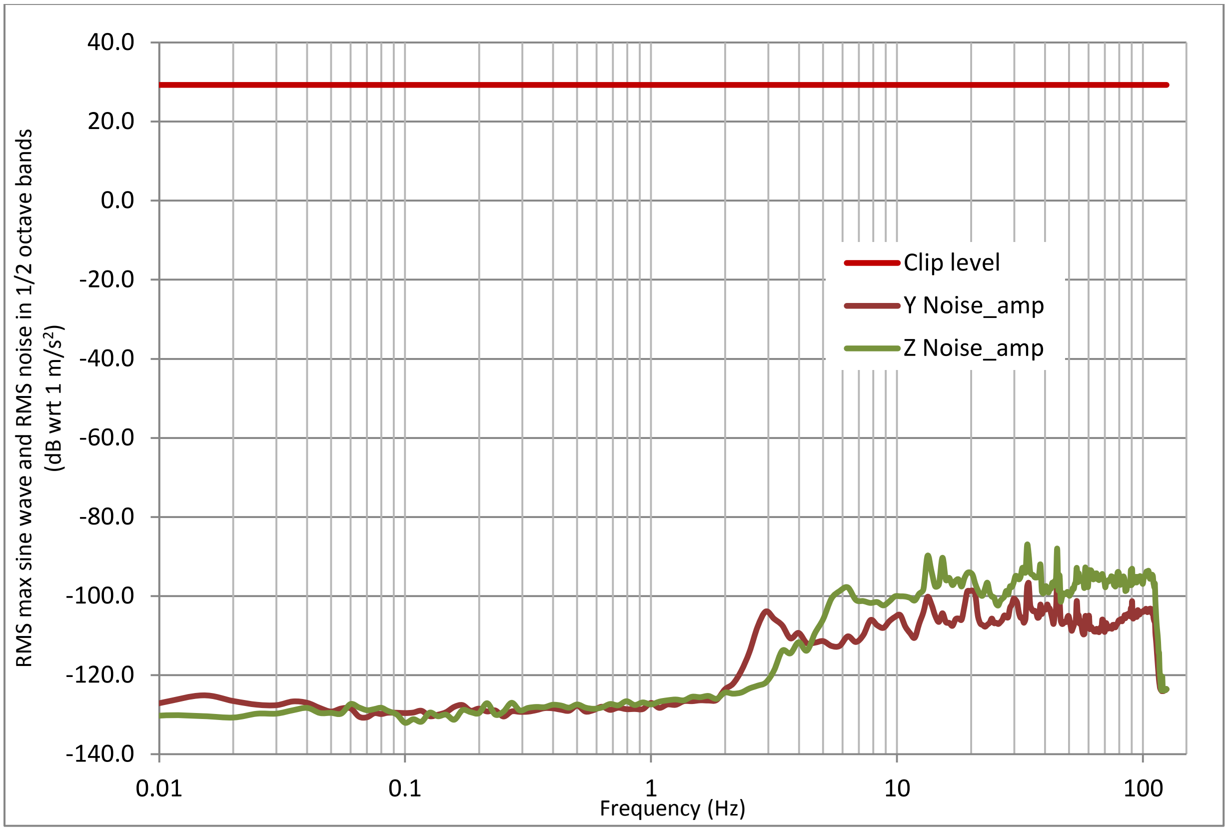

Figure 2 - Comparing RMS Clip SIne Wave to RMS Noise in 1/2 Octave Bands

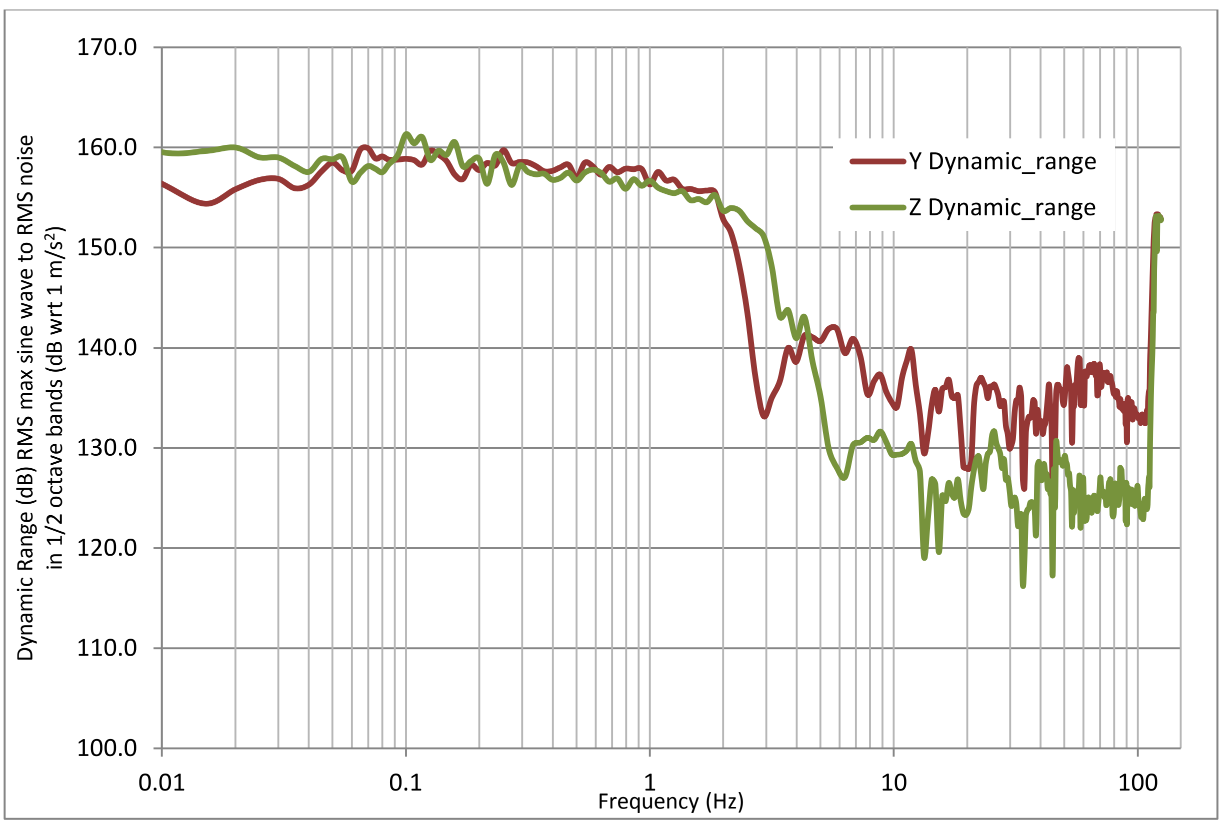

Figure 3 - Dynamic Range Plot

Figures 4 through 6 show how an estimate of noise amplitude and dynamic range can be created by interpolating, when the instrument self-noise is apparent above and below a frequency band where site noise dominates. Because the Y channel shows instrument self-noise both above and below the 2-90 Hz band where site noise dominates, we can interpolate an estimated self-noise curve connecting these lower and upper regions. Figure 4 shows the same PSD as in Figure 1 for the horizontal Y channel, but with a section interpolated for the 2-90 Hz band to estimate the real instrument self-noise in this region. Collecting data from multiple collocated instruments and using a coherence method to subtract away coherent signal (signal common to all instruments) would allow the removal of site noise and reveal the instrument's actual self-noise.

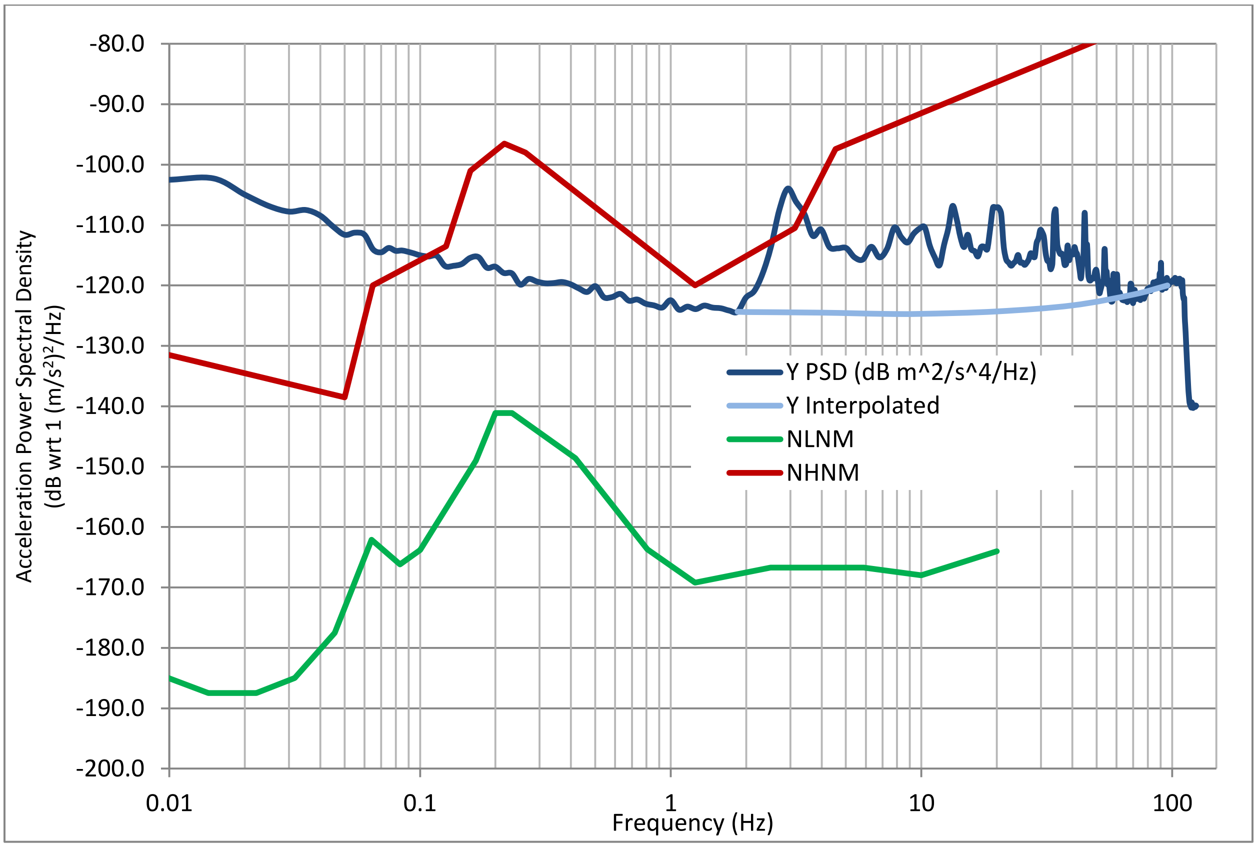

Figure 4 - PSD Self-Noise Showing an Interpolated Y channel Below the Site Noise in the 2-90 Hz Band

Figure 5 shows the calculated equivalent noise amplitude using this interpolated Y data, and Figure 6 shows the dynamic range calculated from that result. While the best method is to test at a sufficiently quiet site or to use coherence analysis with multiple instruments to reveal instrument self-noise, the interpolation technique can provide a useful estimate if the site noise only dominates a small region of the instrument's bandwidth. This is easier for high clip level modes such as 4g or 2g ranges, but higher sensitivity modes such as 1g, 05.g, or 0.25g have lower self-noise floors that are less likely to be visible above site noise.

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August, 2013

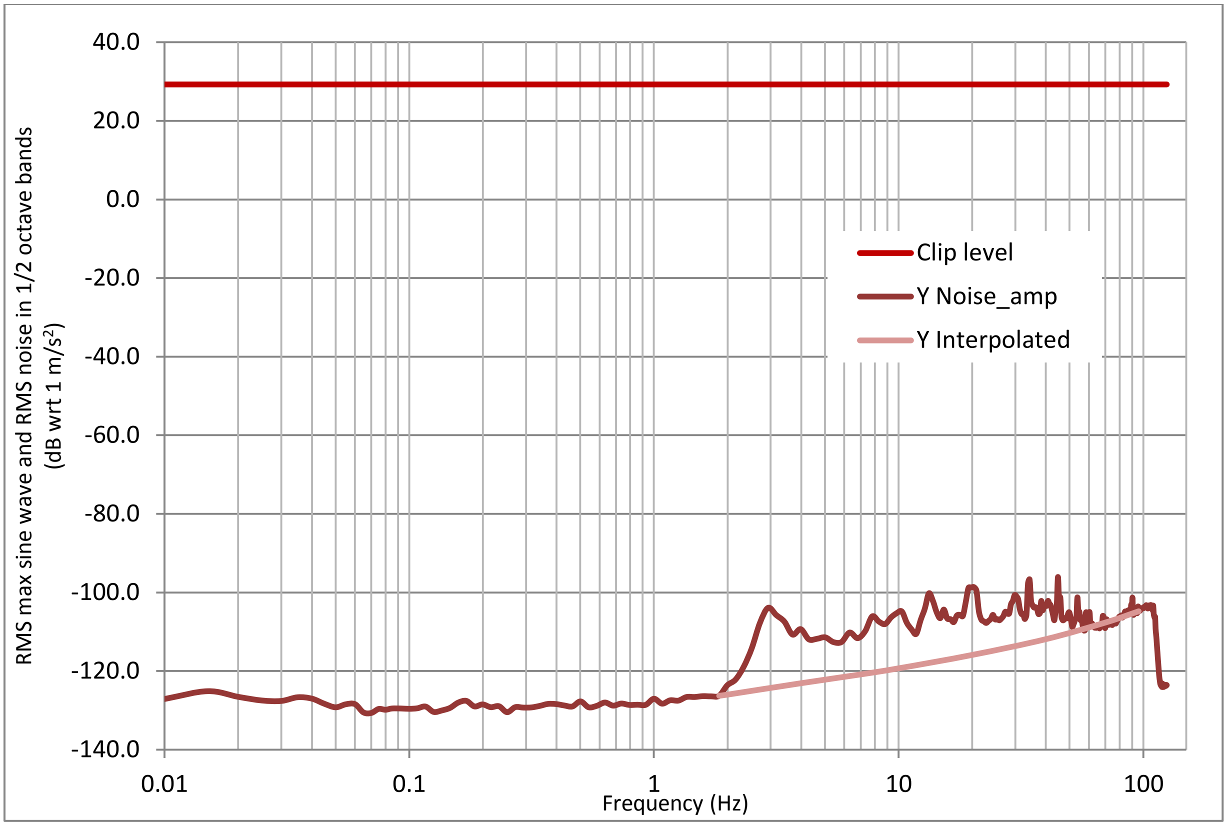

Figure 5 - Comparing RMS Clip SIne Wave to RMS Noise in 1/2 Octave Bands using Interpolated Y Data

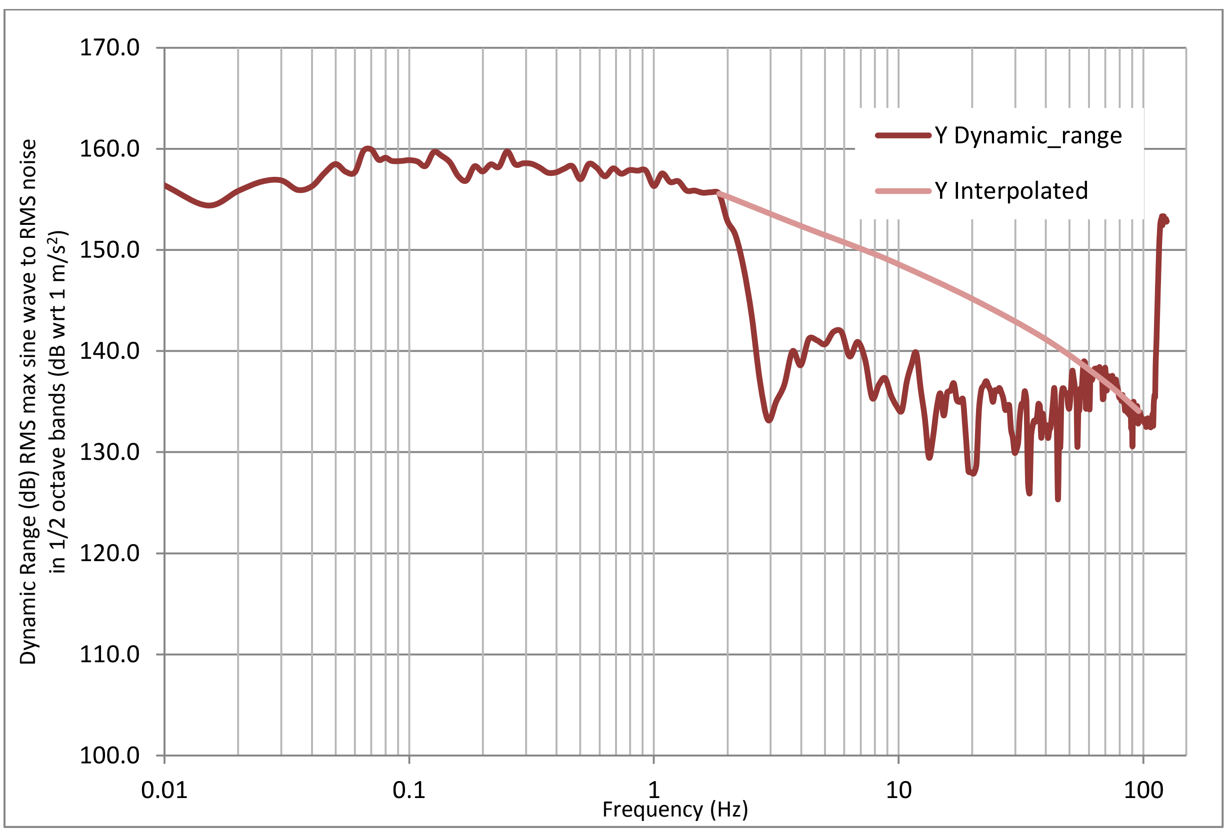

Figure 6 - Dynamic Range Plot using Interpolated Y Data